Third Grade Mathematics

Instructional Focus Documents

Introduction:

The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

Evidence of Learning Statements:

The evidence of learning statements are guidance to help teachers connect the Tennessee Mathematics Standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee Mathematics Standards. These statements are divided into four levels. These four levels are designed to help connect classroom assessments with the performance levels of our state assessment. The four levels of the state assessment are as follows:

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 4: Performance at these levels demonstrates that the student has an extensive understanding and expert ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in the same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting. The provided evidence of learning statements are examples of what students will most likely be able to do and do not represent an exhaustive list.

Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

Operations and Algebraic Thinking (OA)

Standard 3.OA.A.1 (Major Work of the Grade)

Interpret the factors and products in whole number multiplication equations (e.g., 4 x 7 is 4 groups of 7 objects with a total of 28 objects or 4 strings measuring 7 inches each with a total of 28 inches).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Provide inconsistent, inaccurate interpretations of the elements within a multiplication equation. Students at this level may confuse the meaning of the operation of multiplication with the meaning of the other operations.	Choose an accurate interpretation of the factors and product in a given whole number multiplication equations within 100 from options involving the mathematical language of groups and objects. Choose an accurate interpretation of the factors and product in a given whole number multiplication equations within 100 from given simple contextual situations.	Interpret the factors and product in a given whole number multiplication equations within 100 using the mathematical language of groups and objects. Interpret the factors and product in a given whole number multiplication equations within 100 using a simple contextual situation.	Create a contextual situation representing a multiplicative relationship and provide the equation represented within the situation. Explain the relationship that exists between finding the total number of objects when working with groups of objects and the operation of multiplication. Explain the relationship that exists between interpreting factors and products in whole number multiplication equations and skip-counting.

Instructional Focus Statements

Level 3:

Grade 3 is the first formal introduction of multiplication for students. It is crucial that students develop an understanding of the meaning of the operation as well as an understanding of how to compute using multiplication. The instructional focus for this standard should hinge on helping students understand interpreting the factors as groups of objects and the product as the total number of objects represented. Instruction should build on understanding from standard 2.OA.C.4 where students used repeated addition to find the total number of objects arranged in rectangular arrays. Students should discover that multiplication is a more efficient way to calculate a total when considering groups of equal objects.

As students are working to solidify their understanding, they should be given a wide variety of opportunities to analyze equal groups in both arrays and scattered configurations identifying which factors represent the number of groups and which factors represent the size of the group. Students should also be able to articulate that the product describes the total number of objects represented.

Instruction should intentionally connect to standard 3.OA.A.3 as the concept of multiplication as grouping becomes evident when students work with contextual problems.

Level 4:

Instruction at this level should focus on students clearly articulating the meaning of multiplication as finding the total number of objects represented by groups of equal size and explaining why equal sized groups are crucial. Additionally, students can create their own contextual problem representing a multiplicative relationship and explain the factors and product as they relate to the context.

Standard 3.OA.A.2 (Major Work of the Grade)

Interpret the dividend, divisor, and quotient in whole number division equations (e.g., 28 ÷ 7 can be interpreted as 28 objects divided into 7 equal groups with 4 objects in each group or 28 objects divided so there are 7 objects in each of the 4 equal groups).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Provide inconsistent, inaccurate interpretations of the elements within a division equation. Students at this level may confuse the meaning of the operation of division with the meaning of the other operations.	Interpret the dividend, divisor, and quotient from a given division equation in either a partitive or quotitive manner. Represent the interpretation with pictures and provide either a written or verbal explanation of the interpretation.	Interpret the dividend, divisor, and quotient from a given division equation in both a partitive and quotitive manner. Represent both interpretations with pictures and provide either a written or verbal explanation of both interpretations.	Create a contextual problem to represent a given division equation representing a partitive division situation (where the divisor represents the number in each group). Create a contextual problem to represent a given division equation representing a quotitive division situation (where the divisor represents the number of groups).

Instructional Focus Statements

Level 3:

The focus of instruction should be on helping students discover that there are two ways to interpret any division equation. In one interpretation the quotient indicates the number of groups while the divisor represents how many members should be in each group (quotitive division) and in the other the quotient indicates the number of members in each group while the divisor represents the number of groups formed (partitive division). This conceptual understanding should be developed alongside students solidifying their understanding of multiplication as the two types of division are a direct result of the relationship that exists between multiplication and division emphasized in standard 3.OA.B.6. It is important that students explore division to create this dual understanding of the operation as it will impact their understanding of the operation in all subsequent courses. Students should be equally exposed to both types of division throughout grade 3.

As students are developing their conceptual understanding of the two types of division, integration with standard 3.OA.A.3, where students work with contextual problems, can help them more clearly see the need for both types as contextual problems will either illicit partitive or quotitive division but not both. Further, students will be specifically modeling the mathematics occurring in the contextual problem as they find the solution. In order to accurately model the mathematics, students must be able to discern between the two types of division creating a need to understand both. Through contextual problems, students also develop an understanding that "groups" can come in many forms such as packs, bags, bundles, rooms, and boxes. Making sense of what term represents the "group" in the context can be beneficial as students work to develop their conceptual understanding of division as well as determining which type of division they should model.

As students are interpreting division equations, it is imperative that they are given the opportunity to articulate both meanings of division through direct modeling, pictures, written explanations, and verbal explanations.

Level 4:

As students solidify their understanding of the two interpretations of division, they should be able to create a partitive contextual problem that can be solved by a provided division equation and a quotitive contextual problem that could be solved by the same provided division equation. Additionally, students should be able to verbalize specifically why their created problem can be solved by the equation, provide a model of the solution for their created problem, and justify why both the contextual problem and the model represent partitive or quotitive division.

Standard 3.OA.A.3 (Major Work of the Grade)

Multiply and divide within 100 to solve contextual problems, with unknowns in all positions, in situations involving equal groups, arrays, and measurement quantities using strategies based on place value, the properties of operations, and the relationship between multiplication and division (e.g., contexts including computations such as 3 x ? = 24, 6 x 16 = ?, ? ÷ 8 = 3, or 96 ÷ 6 = ?) (See Table 2 - Multiplication and Division Situations).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Choose an equation that matches a simple *unknown product* one-step contextual problem for equal group problem types. Choose a division equation that matches a simple *number of groups unknown* one-step contextual problem for equal group problem types. Choose a division equation that matches a simple *group unknown* one-step contextual problem for equal group problem types.	Multiply within 100 to solve *unknown product* one-step contextual problems for situations involving equal groups, arrays, or comparison problem types utilizing a wide variety of concrete strategies. Divide within 100 to solve *number of groups unknown* or *group unknown* one-step contextual problems for situations involving equal groups, arrays, or comparison problem types utilizing a wide variety of concrete strategies.	Multiply within 100 to solve a wide variety of one-step contextual problems with unknowns in all positions for situations involving equal groups, arrays, and area as problem types utilizing multiple strategies. Divide within 100 to solve a wide variety of *group unknown* one-step contextual problems for situations involving equal groups, arrays, and area as problem types utilizing multiple strategies. Divide within 100 to solve a wide variety of *number of groups unknown* one-step contextual problems for situations involving equal groups, arrays, and area as problem types utilizing multiple of strategies.	Create a one-step *unknown product* contextual problem when given a multiplication equation. Articulate why the contextual situation created matches the provided equation. Create a one-step *number of groups unknown* contextual problem when given a multiplication or division equation. Articulate why the contextual situation created matches the provided equation. Create a one-step *group unknown* contextual problem when given a multiplication or division equation. Articulate why the contextual situation created matches the provided equation.

Instructional Focus Statements

Level 3:

Instruction for standard 3.OA.A.3 should focus on students being able to identify multiplication and division embedded in contextual problems and then solve problems using strategies appropriate to their level of understanding. It is important that instruction connect this standard to standards 3.OA.A.1 and 3.OA.A.2 as these two standards help students develop an understanding of the meaning for each operation. Without a conceptual understanding of the meaning of operations it will be very difficult for students to solve contextual problems. It is important to note that teaching key words does not help students to develop an understanding of these situations. Rather, by using concrete models and drawing pictures, students can relate their actions to whether the situation calls for multiplication or division. Further, instruction should focus on what is known and what is being determined and not on tricks for problem solving such as CUBE or other acronyms that do not promote sense-making.

As students work to solve a wide variety of problems developed to represent common multiplication and division situations, initially they should be employing direct modeling techniques. Over time, modeling techniques will shift to invented strategies focused on place value, the properties of operations, and the relationship between multiplication and division.

In grade 3 students should be working with contextual problems involving *Equal Groups, Arrays, and Area* with unknowns in all positions. The table for common multiplication and division situations is located on page 43 in the Tennessee Mathematics Standards located here. It is important to note that *Compare* situations are not an expectation until grade 4 as they mirror the content in standard 4.OA.A.1.

One final clarification, products within 100 means more than just the multiplication basic facts. It is important to note that this standard works with all products within 100. While this includes the "basic" multiplication facts, it also includes work with factors outside of 0-9. For example, 14 X 4= 56 is not a "basic" fact but is a computation that students in third grade should be able to perform. This should be connected to the meaning of multiplication as 14 groups of 4 or 4 groups of 14 and represented by a model the student understands. Particularly, students should be exposed to an area model representation. This understanding is essential for fourth grade work with 4.NBT.B.5 where students perform multi-digit by one-digit multiplication through the use of the area model to show the distributive property.

Level 4:

Students at this level should be challenged to explain their thinking using multiple representations and make connections between the visual representations and the problem represented as an equation. As an extension, students should be able to create their own contextual problem and explain the solution. When doing so, students should use visual presentations, equations, and precise mathematical vocabulary. By requiring students to create their own contextual problem or competing or conducting an error analysis it requires them to think critically about how or why you would solve a problem using a given equation.

Standard 3.OA.A.4 (Major Work of the Grade)

Determine the unknown whole number in a multiplication or division equation relating three whole numbers within 100. For example, determine the unknown number that makes the equation true in each of the equations: 8 x ? = 48, 5 = ? ÷ 3, 6 x 6 =?

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Choose the unknown product for a given multiplication equation. Choose the unknown quotient for a given division equation.	Determine the unknown product in a multiplication equation using concrete representations. Determine the unknown quotient in a division equation using concrete representations.	Determine the unknown factor in a multiplication equation. Determine the unknown dividend or divisor in a division equation.	Determine all possible factor pairs for a given product within 100. Determine all possible dividend divisor pairs for a given quotient within 100.

Instructional Focus Statements

Level 3:

Grade 3 standards 3.OA.A.1, 3.OA.A.2, and 3.OA.A.3 have an instructional focus of students developing an understanding of the meanings of the operations of multiplication and division and then applying those understandings as they solve multiplication and related division problems using appropriate modeling techniques. Instruction for standard 3.OA.A.4 should focus on students developing and solidifying their understanding of symbolic representations and writing equations. Students should be given multiple opportunities to explore writing equations with missing factors as well as finding the unknown in an equation. It is important to note that the focus is not on memorizing a fact family and knowing "who is in the family or missing from the family". Instruction should focus on the meaning of the factors and quotient/divisor and the product/dividend.

Level 4:

As students become proficient with finding a missing element in a multiplication or division equation, they should be challenged to identify two missing elements. Further, they could be challenged to identify all factor pairs that generate a given product. Likewise, they can be challenged to determine all possible dividend divisor pairs that generate a given quotient. These scenarios develop flexibility of thinking for students as they are finding two unknowns as opposed to one unknown.

Standard 3.OA.B.5 (Major Work of the Grade)

Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known (Commutative property of multiplication). 3 x 5 x 2 can be solved by (3 x 5) x 2 or 3 x (5 x 2) (Associative property of multiplication). One way to find 8 x 7 is by using 8 x (5 + 2) = (8 x 5) + (8 x 2). By knowing that 8 x 5 = 40 and 8 x 2 = 16, then 8 x 7 = 40 + 16 = 56 (Distributive property of multiplication over addition).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose equivalent expressions that illustrate the commutative property. Decompose a number into a sum of its parts.	Choose equivalent expressions that illustrate the associative property. Choose equivalent expressions that illustrate the distributive property.	Apply the commutative property as a strategy to multiply. Apply the associative property as a strategy to multiply. Apply the distributive property as a strategy to multiply over addition. Indicate that the commutative and associative properties do not apply to the operation of division.	Explain why the commutative and associative properties do not apply to the operation of division. Apply the commutative property as a strategy to multiply and explain why it works. Apply the associative property as a strategy to multiply and explain why it works. Apply the distributive property as a strategy to multiply over addition and explain why it works.

Instructional Focus Statements

Level 3:

The focus of this standard is to explore the properties in relation to multiplication. It is equally important that students explore why properties do not apply to division. Explorations should focus on the commutative, associative, and distributive properties. It is important to note that students need not know the formal terms for the properties.

As students interact with the commutative property, arrays are a particularly helpful tool as they allow students to easily manipulate and see what happens when they interchange the number of rows and columns.

As students explore and discuss various solution paths for finding the product of three factors, they discover that some combinations make for much more efficient solutions. Students may also see that factors can be decomposed in order to associate them. For example, 8 x 6, could decomposed into 2 x 4 x 6 and then compute (6 x 4) x 2 as a more friendly computation. Students should be encouraged to look for ways to decompose numbers that make the most sense to them. They should then be asked to explain and defend their reasoning. Additionally, the associative property should be connected in standard 3.NBT.A.3 where students multiply one-digit whole numbers by multiples of 10 (e.g., 9 x 80= 9 x 8 x 10).

The distributive property should be explored in the context of composing and decomposing factors into a sum of its parts. Grade 3 students should be challenged to use the distributive property when working with larger, less familiar factors such as 14. For example, 14 X 4 can be represented as (10 + 4) x 4 and then (10 x 4) + (4 x 4). This allows students to decompose 14 into more familiar products supporting standard 3.OA.A.3. Students will need ample opportunity to interact with concrete representations that they manipulate to form simpler problems. This type of reasoning is critical for developing fluency in students. For example, students use arrays and/or area models to show the breakdown of the problem using the distributive property and then connect that model to the equation.

Ultimately, this standard supports students to develop a strong sense of flexibility with numbers to help them realize that there are multiple ways to solve problems.

Level 4:

As students solidify their understanding of the various properties, they should be challenged to mathematically explain why the properties work with multiplication but not with division. Additionally, they should be challenged to mathematically justify why all 3 properties (commutative, associative, and distributive) are valid. Students can also be challenged to justify when they feel each property is most useful.

Standard 3.OA.B.6 (Major Work of the Grade)
Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose an unknown factor equation that could be used to represent a given division expression.	Restate a division expression as an unknown factor equation and find the solution using a visual model.	Restate a division expression as an unknown factor equation, solve the equation, and explain the relationship that exists between multiplication and division.	Create a division expression, restate it as an unknown factor equation, and solve the equation. Explain why the simplified expression and the solution to the equation are the same. Explain the mathematical relationship that exists between multiplication and division,

Instructional Focus Statements

Level 3:

As students are solidifying their understanding of the meaning of multiplication and division (standards 3.OA.A.1, 3.OA.A.2, and 3.OA.A.4), it is equally important that they understand the relationship that exists between the two operations. The instructional focus for this standard should be on helping students understand how the concept of “groups” is relevant to both multiplication and division. Expounding upon this understanding can lead students to understand how division can be seen as an unknown factor problem. As with the other standards in the Operations and Algebraic Thinking domain, it is imperative that the standards be connected to one another for students. Further, this standard is a crucial building block in helping students become fluent in division within 100.

In working with this standard, students need a wide variety of experiences where they are asked to identify what information is known and what they are trying to find. While this standard does not explicitly call for a contextual situation, students benefit from contextual situations as they are developing an understanding of the relationship between multiplication and division. Contextual problems allow students the opportunity to use concrete materials and draw pictures while framing numbers with a concrete situation.

Level 4:

Students at this level of understanding should be challenged to verbalize why division problems can be solved as unknown-factor problems. They should be able to mathematically explain the relationship that exists between multiplication and division and further why that relationship allows for division to be solved as an unknown-factor problem.

Students can be challenged to create a division expression, restate it as an unknown factor equation, and solve the equation. They should be able to explain why the simplified expression and the solution to the equation are the same.

Standard 3.OA.C.7 (Major Work of the Grade)

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of 3rd grade, know from memory all products of two one-digit numbers and related division facts.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Multiply and divide within 100 using concrete strategies. Inconsistently multiply when one factor is 1, 2, 5, or 10 using mental strategies. Students can sometimes produce answers using strategies relating multiplication and division or properties of operations without recording their thinking on paper. Inconsistently divide when the divisor is 1, 2, 5, or 10 using mental strategies. Students can sometimes produce answers using strategies relating multiplication and division or properties of operations without recording their thinking on paper.	Fluently multiply when one factor is 1, 2, 5, or 10 using mental strategies. Students are consistent in their ability to efficiently and accurately produce answers using strategies relating multiplication and division or properties of operations without recording their thinking on paper. Fluently divide when the divisor is 1, 2, 5, or 10 using mental strategies. Students are consistent in their ability to efficiently and accurately produce answers using strategies relating multiplication and division or properties of operations without recording their thinking on paper. Inconsistently multiply when one factor is 3, 4, 6, 7, 8, or 9 using mental strategies. Students can sometimes produce answers using strategies relating multiplication and division or properties of	Know from memory all products of two single-digit numbers and their related division facts. Fluently multiply and divide within 100 using strategies relating multiplication and division or properties of operations.	Fluently multiply and divide within 100 using mental strategies. Students are consistent in their ability to efficiently and accurately produce answers without recording their thinking on paper. Students can explain or defend their answer in multiple different ways.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	operations without recording their thinking on paper. Inconsistently divide when the divisor is 3, 4, 6, 7, 8, or 9 using mental strategies. Students can sometimes produce answers using strategies relating multiplication and division or properties of operations without recording their thinking on paper.		

Instructional Focus Statements

Level 3:

As stated in the introduction of the Tennessee Mathematics Standards, fluency is the ability to apply procedures accurately, efficiently, and flexibly. Fluency is about a student being able to flexibly think about the problem posed in order to efficiently answer by employing a strategy from their tool box that makes sense in that particular situation leading to an accurate answer. Fluency is not defined by speed. There is no one strategy that works every time for every student. Each child develops fluency from the strategies that individually work best for them. The natural progression of learning begins with direct modeling. It is important that students who still need direct modeling in order to grasp the mathematics be allowed to do so. With these students, it will be important over the course of the year to help them move from direct modeling to more strategy based approaches. Ultimately strategy based approaches are what builds fluency for students. Strategies that students may use to become fluent in multiplication include: doubling, composing and decomposing factors to use known facts, and the use of properties. With all of these strategies, students should model and relate the model to a written equation.

As students become more fluent with multiplying and dividing numbers, they should start to produce answers without recording their thinking and explaining their mental thought process. Additionally, students should have many opportunities to practice, explain their thinking, and compare to make connections with multiple strategies. Number Talks, written explanations, and selecting and defending the strategy that makes the most sense to them will allow students to develop conceptual understanding so that they become fluent over time. Knowing from memory all products of two one-digit numbers and related division facts is the culmination of the conceptual understanding of multiplication and division developed from all previous standards in this domain.

Level 4:

As students develop a wider range of mental strategies that they are comfortable with, they should be able to explain the connections that exist between multiple strategies. Additionally, they should be challenge to defend their reasoning as to why a particular strategy is best in a given situation. Students should be able to explain what misconception took place to produce an incorrect answer. It is imperative as students transition to using mental strategies that they are asked questions which press for the underlying mathematics. This allows students to provide an explanation of their thinking using precise mathematical vocabulary.

Standard 3.OA.D.8 (Major Work of the Grade)

Solve two-step contextual problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Solve one- and two-step contextual problems, with unknowns in all positions, involving a wide variety of common addition and subtraction situations.	Solve one-step multiplication or division contextual problems, with unknowns in all positions, involving *Equal Groups, Arrays, and Area* situations. Represent one-step contextual problems with an equation using a letter to represent the unknown quantity. Choose a reasonable answer to a given one-step contextual problem without working the problem.	Solve two-step contextual problems using the four operations involving both addition/subtraction and multiplication/division situations with unknowns in a wide variety of positions. Represent two-step contextual problems using a series of equations with a letter standing in for the unknown quantity. Choose a reasonable answer to a given two-step contextual problem involving multiple operations without working the problem.	Use mental computation and estimation strategies to present a reasonable solution to a given contextual two-step problem. Solve the contextual problem including a representation using equations with a letter standing in for the unknown quantity and compare the original estimation with the actual answer, providing mathematical justification for any discrepancies. Create a contextual problem that represents a given two-step equation (e.g., 2 x 5 + 1= x <u>or</u> m + 4 – 9 =20).

<u>Instructional Focus Statements</u>

<u>Level 3:</u>

In grade 2, students solved one- and two-step addition and subtraction contextual problems with a focus on common addition and subtraction situations with unknowns in all positions. In grade 3, standard 3.OA.A.3 students were first introduced to solving one-step multiplication and division contextual problems. It is important that standard 3.OA.D.8 build on these experiences to extend student thinking as they solve two-step contextual problems using all four operations.

In transitioning all students to working with two-step, multi-operation contextual problems, instruction should initially focus on problems involving smaller, familiar numbers allowing students to focus on the conceptual understanding of multiple operations within the problem as opposed to focusing on computation with less familiar numbers. Additionally, it is easier for students to begin with problems that call for commonly paired operations (i.e., addition/subtraction, multiplication/division) within the problem and then move to working with two-step problems that involve less common pairings (e.g., addition and division).   It is important to call out that students should continue to use manipulatives, multiple strategies, and written equations when solving two-step contextual problems. To demonstrate their understanding, they should be able to explain the connections between the visual representation and the equation(s) that represents the problem. Additionally, students should be encouraged to use multiple strategies and make connections between each strategy. For example, students may write individual equations for each step in a two-step problem or write both steps in one equation. This is a good opportunity for students to compare their work to others and explain why both are correct or in some cases incorrect and explain the connection between the two strategies. The instructional focus should be more on students understanding two-step problems and sense making as opposed to simply getting a correct answer.

Teaching key words to associate with addition, subtraction, multiplication, and division should not be an instructional focus. Instruction should focus on developing an understanding of what operation is needed to solve the problem rather than focusing on key words that sometimes, but not always, associate with the operation.

Instruction should also focus on encouraging students to assess the reasonableness of their answers. Students should use estimation strategies and mental computations as they consider reasonableness. One beneficial instructional strategy is for students to estimate a solution prior to solving the problem.

Level 4:

As students deepen their understanding of two-step contextual problems, they should be able to represent these problems with a mathematical drawing, diagram, and an equation with a letter for the unknown number. They should be able to explain their thinking using multiple representations and make connections between the visual representations and their equations. Students should be able to use mental computation and estimation strategies to present a reasonable solution, solve the problem, and then compare the original estimation and the actual answer providing mathematical justification for any discrepancies.

Additionally, students should be able to create their own two-step contextual problem and explain the solution. When doing so, students should use visual representations, equations, and precise mathematical vocabulary.

Standard 3.OA.D.9 (Major Work of the Grade)

Identify arithmetic patterns (including patterns in the addition and multiplication tables) and explain them using properties of operations. For example, analyze patterns in the multiplication table and observe that 4 times a number is always even (because 4 x 6 = (2 x 2) x 6 = 2 x (2 x 6), which uses the associative property of multiplication) (See Table 3 - Properties of Operations).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Fill in missing elements in a simple pre-identified mathematical pattern. Extend a pre-identified mathematical pattern.	Identify arithmetic patterns in addition and explain the pattern using properties of operations. Explain how a pre-identified mathematical pattern works using properties of operations.	Identify arithmetic patterns that exist in mathematics and explain the patterns using properties of operations.	Identify complex arithmetic patterns that exist in mathematics and explain the patterns using properties of operations. Identify multiple arithmetic patterns that exist in mathematics and explain the relationship between the patterns using properties of operations.

Instructional Focus Statements

Level 3:

In many instances, patterns are a way in which students connect with the world around them. They have looked at and extended patterns as a way to build spatial recognition skills since they were very young. Patterns are one way in which children look for and find mathematical beauty in and make sense of the world around them. Instruction for this standard should focus on students using that same level of curiosity to discover a wide variety arithmetic patterns that exist in mathematics and then dig deeper to explain mathematically why the pattern occurs. It is important to note that this standard is not limited to addition and multiplication. Students should be exploring all four operations looking for patterns that exist and searching for justification as to why the pattern occurs.

In grade 2, students explored patterns of odd and even numbers in a hundreds chart. In grade 3, students should be looking at a wide variety of generated numbers such as those in addition, subtraction, multiplication, and division tables. It is crucial that students be the ones making sense of and discovering arithmetic patterns as opposed to being directed to them. For example, they may discover patterns that exist in pairs of numbers that yield the same sum in an addition table or discover that every third number in a multiplication table is a multiple of three. It is crucial that students not stop simply with the discovery of a pattern, but that they investigate mathematically why the pattern occurs.

As many mathematical concepts build on patterns and students ability to identify those patterns, the more experience children have with looking for and making sense of patterns the better prepared that they will be to access concepts requiring patterned thinking in subsequent grades. As students notice and wonder about the patterns they see in the tables and in number sequences, continue to ask questions such as, " How do you know?" and "Does that always work?" to push their thinking about patterns.

Level 4:

As students develop a deeper conceptual understanding of patterns and the mathematics behind them, they should be able to identify increasingly more complex arithmetic patterns that exist in mathematics and explain the patterns using properties of operations. Additionally, they should be challenged to identify multiple arithmetic patterns that exist is and explain the relationship between the patterns using properties of operations. The ability to identify patterns is critical as students begin to think algebraically in subsequent courses.

Numbers and Operations in Base Ten (NBT)

Standard 3.NBT.A.1 (Supporting Content)
Round whole numbers to the nearest 10 or 100 using understanding of place value.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Given a two-digit number plotted on a pre-partitioned, pre-labeled number line, round to the nearest 10. Determine the two tens that a two-digit number falls between. Determine the two hundreds that a three-digit number falls between.	Determine the two tens that a three-digit number falls between. Use place value understanding and a visual representation to round a two-digit whole number to the nearest 10.	Use place value understanding to round whole numbers (up to 1,000) to the nearest 10 or 100. Explain the connection that exists between rounding and distance.	Use place value understanding to round whole numbers (greater than 1,000) to the nearest 10 or 100. Provide justification for why rounding a whole number to the nearest 10 as opposed to the nearest 100 would be a better choice in a given situation. Conceptually explain how to round a whole number to the nearest 10 or 100 using place value understanding.

Instructional Focus Statements

Level 3:

Grade 3 is the first time that students have encountered the concept of rounding. Thus, the instructional focus should center on allowing students a wide variety of instructional opportunities so that they build a conceptual understanding of what it means to round. Students need to discover the connection that exists between rounding and distance prior to thinking about rounding through the lens of place value. The use of hundred charts and number lines will be imperative in developing rounding understanding particularly as students connect the concepts of rounding and distance. Students should be challenged to think about what happens with distance when the number being rounded is exactly halfway between two tens or two hundreds (e.g., 55). This poses a great opportunity for fostering discourse around if the number should be rounded to 50 or 60 in this case.

Once students have moved beyond the need for concrete representations, they should begin examining the relationship between place value and rounding. Students should begin by rounding numbers to the nearest ten. They should be encouraged to consider the value of the digits in the ones place and making connections as to how that value is related to the distance from the number being rounded to the ten on either side of the number. At this point, students should begin making their own generalization around how the value of the number in the ones digit effects rounding to the nearest 10 providing justifications for the generalizations based on the concept of distance. Similarly, students should examine how the digit in the tens place effects rounding a number to the nearest hundred coupled with a conversation around why the tens place makes a significant difference but the ones digit doesn't.

It is important to note that the rounding "rules" can create a variety of misconceptions for students. For example, "rounding down" leads students to believe the digit in the tens place would decrease by one when in reality it remains the same. It is imperative that this standard not become a procedure without connections to the mathematics behind rounding.

Level 4:

As students grasp the concept of rounding to the nearest ten or hundred within 1,000, they can be challenged to work with rounding numbers greater than 1,000. It is important that as students expand the number set they are working with that they do not loose site of the conceptual understanding of rounding. Additionally, Students should be challenged to apply their understanding of rounding as it applies to real -world situations. For example, If you had 62 coming to a party and you wanted to buy enough food would it be better to round your number to 60, 70, or 100 and explain why. This type of questioning allows students to see a real application of the value of rounding.

Standard 3.NBT.A.2 (Supporting Content)
Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add and subtract two-digit numbers using visual models. Mentally add or subtract multiples of 10 or 100 from a given number.	Add or subtract numbers within 1,000, including composing or decomposing as needed, using visual models. Apply the commutative or associative property to solve problems.	Add and subtract within 1,000, including composition and decomposition as needed using strategies and algorithms. Write an equation to show the relationship between addition and subtraction.	Explain the method used in finding the sum or difference. Identify an error in an addition and subtraction problem, correct the error, and explain the reasoning of the correct solution path and identify the misconception that caused the initial error. Add and subtract groups of three or more numbers including both two and three-digit numbers which elicit composition and decomposition of whole numbers using strategies and algorithms.

Instructional Focus Statements

Level 3:

In grades 1 and 2, students developed strategies to add and subtract within 100. In standard 3.NBT.A.2, students should build on and extend that understanding to add/subtract fluently with numbers within 1000. Students should use methods based on place value, properties of operations, the relationships between addition and subtraction, and algorithms. Students should also have opportunities to explore methods that can be generalized to larger numbers so that these methods can be extended to 1,000,000 in grade 4. Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties.

As students are solidifying their understanding of adding and subtracting within 1000, they should build upon their own "invented strategies" developed in grades 1 and 2. Invented strategies are when students begin eliciting their understanding of composing and decomposing numbers in flexible ways in order to more quickly figure out a computation. Some examples of invented strategies are compensation, counting on, counting back, or making a group of ten.

Students should also be able to subtract within 1000 by viewing a subtraction problem as an unknown addend problem, (e.g., 276 + ? = 425). Also, counting-on and adding-on methods for addition can be used for subtraction. Many students struggle with subtracting numbers that have zero tens and/or zero ones, sometimes referred to as "subtraction across zeros". It is imperative that students conceptually understand how to compose and decompose numbers when regrouping is necessary.

Those strategies are the foundation for fluency with addition and subtraction in grades K-3. Fluency is about recognizing when one strategy or procedure is more appropriate to apply than another so that students are computing with flexibility, efficiency, and accuracy. Students should develop a procedural understanding grounded in their previous conceptual understanding. Instruction should make explicit connections between those strategies and algorithms.

When algorithms are introduced, it is imperative to make connections to place value and to the partial sum or expanded algorithm for addition and/or subtraction. It should be noted that often different invented algorithms are more efficient for particular numbers. For example, 1000- 999 can be more efficiently solved by finding the difference or adding up than it would be to use regrouping. As students work with concrete and representational strategies alongside algorithms, they should be able to explain their reasoning, make connections to different solution paths, and explain the relationships between methods using precise mathematical vocabulary.

Additionally, students should be able to apply addition and subtraction within contextual situations.

Level 4:

As students solidify their procedural and computational fluency in adding and subtracting within 1000, they should be able to use multiple strategies and explain the relationship that exists between strategies and algorithms. Students should also be able to explain when one method is more efficient to them than another method. Students should also be able to assess the reasonableness and accuracy of completed problems (both correct and those containing misconceptions), allowing them to reason about the process used in the solution path.

For further extension, students should be challenged to add and subtract groups of three or more numbers including both two and three-digit numbers which elicit composition and decomposition of whole numbers using strategies and algorithms. Students should also be able to explain their answer using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Additionally, students should be able to explain, using precise mathematical vocabulary, the connections that exists between multiple strategies across both addition and subtraction. Students should be able to construct a viable argument (MP 3) to justify when strategies are more efficient.

Standard 3.NBT.A.3 (Supporting Content)
Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Given a multiple of 10 in the range 10-90, chose the factor that along with 10 would lead to the given product. Multiply single-digit whole numbers by 10 using concrete models (e.g., base ten blocks).	Given a multiple of 10 in the range 120-810, chose factors one of which is a one-digit whole number and the other being a multiple of 10 that would lead to the given product. Multiply single-digit whole numbers by 10 using strategies based on place value. Multiply single-digit whole numbers by multiples of 10 using repeated addition or skip counting.	Multiply single-digit whole numbers by multiples of 10 in the range 10-90 using strategies based on place value. Multiply single-digit whole numbers by multiples of 10 in the range 10-90 using strategies based on properties of operations (e.g., 9 X 80 =9 X (8 x 10) = (9 X 8) X 10 = 72 X 10 = 720).	Multiply single-digit whole numbers by multiples of 10 in the range 10-90 using strategies based on place value and provide justification for their thinking. Multiply single-digit whole numbers by multiples of 10 in the range 10-90 using strategies based on properties of operations and provide justification for their thinking.

Instructional Focus Statements

Level 3:

This standard builds on the understanding students are developing around the meaning of multiplication and the various strategies that are used when multiplying in the grade 3 Operations and Algebraic Thinking (OA) standards. It is important that students have an understanding of the operation of multiplication before engaging in multiplying with multiples of 10. The instructional focus for this standard is around helping students conceptually understand the role of ten and subsequently multiples of ten as they relate to multiplication and connect the operation of multiplication to the base ten number system. This standard is foundational for students as they generalize place value understanding for multi-digit whole numbers in grade 4 and focus on the place value system in grade 5.

Instruction should not focus on adding zeros to the end of a product as this builds a procedural solution with no real connection to what is mathematically occurring. Instruction should be rooted in using base ten language and the use of properties paying particularly close attention to the associative property as students work through understanding the effects of multiplying a single-digit whole number by a multiple of 10. As instruction progresses from conceptual to representational to abstract it is appropriate for students to generalize patterns that they notice when multiplying by 10 or a multiple of 10. It is equally important that their explanations for discovered patterns be rooted in the understanding of the properties of operations and place value.

As instruction progresses from multiplying a single-digit whole number by 10 to a single-digit number by a multiple of 10, the associative property becomes a particularly useful tool for students in developing conceptual understanding. As students are solving problems such as 2 X 60, they should discover through exploration that decomposing 60 into 6 x 10 yielding 2 x (6 x 10) and then utilizing the associative property to yield (2 x 6) x 10 leads them to 12 x 10 which can be interpreted as 12 groups of 10 (standard 3.OA.A.1). Initially students can use manipulatives, skip count, or use repeated addition to find the solution. Over time, students will begin discovering patterns that exist in this process and be able to provide explanations for discovered patterns rooted in the understanding of the properties of operations and place value.

Level 4:

Students with this level of understanding can clearly articulate how to multiply single-digit whole numbers by multiples of 10 in the range 10-90 using strategies based on properties of operation and place value strategies providing justification for their thinking. Additionally, they can compare and contrast various methods, identify which is their preferred method, and justify why. Students at this level can also begin to explore what happens when multiplying a single-digit number by 100 and multiples of 100 in the range 100-900 and then draw comparisons to multiplying a single digit by a multiple of 10.

Numbers and Operations-Fractions (NF)

Standard 3.NF.A.1 (Major Work of the Grade)

Understand a fraction, $\frac{1}{b}$, as the quantity formed by 1 part when a whole is partitioned into *b* equal parts (unit fraction); understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts of size $\frac{1}{b}$. *For example,* $\frac{3}{4}$ *represents a quantity formed by 3 parts of size* $\frac{1}{4}$.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize when a whole is partitioned into 2, 3, 4, 6, or 8 equal shares and use appropriate vocabulary to describe one of the shares. Recognize that equal shares of identical wholes need not have the same shape.	Identify $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts when provided a visual representation. Identify $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$ when provided a visual representation.	Identify $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts. Create visual fraction models to represent a given unit fraction. Identify $\frac{1}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.	Create a variety of visual fraction models to represent the same given unit fraction. Create visual fraction models to represent a fraction $\frac{a}{b}$ accurately.

Instructional Focus Statements

Level 3:

The primary focus of fraction instruction in grade 3 should be centered on two things: 1) developing the understanding of fractions as numbers and 2) developing the idea of fractional amounts as equal parts of the same whole. Thus, collections of objects (set models) should not be a focus of grade 3 instruction. Additionally, in developing the concept that fractions are numeric values, there is not a need to distinguish between “proper” and “improper” fractions. Students should be able to verbalize that equal parts are parts with equal **measure**. That measure may be area or length, depending upon the visual fraction model. When describing a fraction, focus on using correct language. Consider the fraction 5/4; this is described as 5 pieces that are ¼ in size. The most common error is to discuss the fraction as 5 out of 4 which is more of a ratio understanding. The ultimate goal is for students to see unit fractions as the basic building block for all fractions in the same sense that 1 is the basic building block for whole numbers.

Level 4:

Students should be encouraged to not only use provided visual fraction models to represent fractions, but to also create their own representations providing a justification as to why/how their model works and identifying the whole and the equal parts.

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Standard 3.NF.A.2 (Major Work of the Grade)

Understand a fraction as a number on the number line. Represent fractions on a number line.

3.NF.A.2a

Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint locates the number $\frac{1}{b}$ on the number line. *For example, on a number line from 0 to 1, students can partition it into 4 equal parts and recognize that each part represents a length of* $\frac{1}{4}$ *and the first part has an endpoint at* $\frac{1}{4}$ *on the number line.*

3.NF.A.2b

Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line. *For example,* $\frac{5}{3}$ *is the distance from 0 when there are 5 iterations of* $\frac{1}{3}$.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Represent a fraction $\frac{1}{b}$ on a number line diagram on the interval from 0 to 1 when a pre-partitioned number line in b equal parts is provided.	Represent a fraction $\frac{a}{b}$ on a number line diagram on the interval from 0 to 1 when a pre-partitioned number line in b equal parts is provided.	Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has length $\frac{1}{b}$ and that the endpoint locates the number $\frac{1}{b}$ on the number line. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths of $\frac{1}{b}$ from 0 when partitions are not provided on the number line. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.	Create two double number lines where the whole is the same on both number lines but both number lines are partitioned into different unit fractions focusing on unit fractions where the denominators are factors of one another (e.g., halves, fourths, and eighths).

Instructional Focus Statements

Level 3:

Instruction with number lines will reinforce the idea of fractions as numbers for students. By the end of grade 3, students must grasp how to use number lines as a tool to represent fractions. This is imperative so that when students begin modeling operations with fractions in subsequent grades, creating the number line is not a barrier.

Students should develop an understanding that the **interval** from 0 to 1 represents a whole. It is important to emphasize that on a number line, 0 to 1 has the same length as 1 to 2, as 2 to 3, etc. These intervals represent equal wholes the same way that area inside congruent circles or congruent rectangles may represent wholes. It is important to help students see the connection/relationship that exists between number line models and area-based models, such as fraction strips, fraction circles, tape diagrams, etc.

Students should practice marking off number lines into accurate amounts (e.g., thirds). Students should work with a balance of pre-partitioned number lines and number lines that the student must partition. Graph paper can serve as a helpful tool for students as they develop this skill.

Additionally, it is important to continue the analogy between fractions and whole numbers as students create number lines. Thinking about a number line, just like 5 is marking off five 1s, $\frac{5}{4}$ is marking off five $\frac{1}{4}$ s.

Level 4:

At this level students evolve from thinking about only one unit fraction at a time on one number line, to having double number lines marked off using different unit fractions. These number lines will help students begin to compare fractional values. Additionally, graph paper can serve as a helpful tool for students as they develop this skill. That said, it is imperative that teachers explain what the whole is being represented by and also that the width between the marks on the graph paper may represent different unit fractions. This idea is often cognitively difficult for students to grasp.

Standard 3.NF.A.3 (Major Work of the Grade)
Explain equivalence of fractions and compare fractions by reasoning about their size.
3.NF.A.3a Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.
3.NF.A.3b Recognize and generate simple equivalent fractions and explain why the fractions are equivalent using a visual fraction model.
3.NF.A.3c Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. *For example, express 3 in the form* $3 = \frac{3}{1}$; *recognize that* $\frac{6}{1} = 6$; *locate* $\frac{4}{4}$ *and 1 at the same point on a number line diagram.*
3.NF.A.3d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Determine if two fractions less than 1 are equal given that one of the fractions is a unit fraction when provided a visual fraction model. Generate fractions equivalent to 1. Identify that two fractions that are both equivalent to 1 are equal. Locate a fraction equal to 1 accurately at 1 on a number line.	Determine if two fractions less than 1 are equal when provided a visual fraction model. Express a whole number as a fraction with a denominator of 1. Identify simple equivalent fractions when provided a visual fraction model. Compare two fractions with like numerators or like denominators when a visual fraction model is provided using appropriate symbols.	Explain what it means for two fractions to be equal. Determine if two fractions are equal including cases where the fractions may have values greater than 1. Generate simple equivalent fractions. Compare two fractions with like numerators or like denominators using appropriate symbols.	Create a problem to represent when two seemingly equivalent fractions would not be equal and explain why (e.g., provide a situation when $\frac{2}{4}$ would not equal $\frac{1}{2}$ and explain why). Express whole numbers as fractions with denominators other than 1 (e.g., $2=\frac{8}{4}$). Explain why when given two fractions with like numerators, the fraction with the larger denominator is the smallest. Order more than two fractions with like numerators or like denominators from least to greatest or greatest to least using

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
			appropriate symbols, and provide a justification.

Instructional Focus Statements

Level 3:

Instruction should focus on students developing a conceptual understanding of how to compare fractions using mathematical reasoning. Students should be experimenting with a variety of visual fraction models (area models and number lines) in order to develop a strong understanding of equivalent fractions as opposed to using algorithms and procedures. The understanding of equivalent fractions includes fractions equal to whole number values.

Students compared lengths in grade 2. They should extend their prior knowledge of comparing length in order to develop an understanding of comparing fractions with like denominators and fractions with like numerators. Number lines are a natural place for students to make this connection. Greater length on a number line means a greater fraction. Students should be challenged to compare not only proper fractions but a balance of proper and improper fractions.

A common misconception for grade 3 students is that a unit fraction with a larger denominator is larger than a unit fraction with a smaller denominator. Students need opportunities to compare these using a variety of visual fraction models so that students discover the flaw in this way of thinking.

Level 4:

The instructional focus should evolve so that students are making generalizations for how to compare fractions with sound mathematical reasoning guiding their justifications. Comparing fractions often becomes a place where “tricks” for quick, procedurally gained answers are employed.

Students should supply valid, conceptually-based justifications for their thinking processes. One such extension here is as students think deeply about fractions as numbers represented on a number line, they develop a sense of order in terms of position. It is important that students develop a conceptual understanding that a number to the right on the number line is larger and that the number to the left is smaller and that the student is able to supply mathematical justification.

Measurement and Data (MD)

Standard 3.MD.A.1 (Major Work of the Grade)

Tell and write time to the nearest minute and measure time intervals in minutes. Solve contextual problems involving addition and subtraction of time intervals in minutes. *For example, students may use a number line to determine the difference between the start time and the end time of lunch.*

Note: As unit conversions are not within grade 3, it is not necessary for students to convert lengths of time over 60 minutes into hours and minutes.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Tell and write time in half and quarter hours. Tell and write time to the nearest 5 minutes. Solve elapsed time unknown problems where both the start time and end time are within the same hour when a visual representation (i.e. clocks or a number line) is provided.	Tell and write time to the nearest minute. Solve elapsed time unknown problems where both the start time and end time are within the same hour. Solve start time unknown problems where the start time and the end time are within the same hour. Solve end time unknown problems where the start time and the end time are within the same hour.	Solve elapsed time unknown problems where the time interval spans across a change in hour (i.e. find the time interval from 3:45 p.m. to 4:15 p.m.). Solve start time unknown problems where the time interval spans across a change in hour. Solve end time unknown problems where the time interval spans across a change in hour. Solve contextual problems involving addition and subtraction of time intervals in minutes.	Solve multi-step contextual problems involving addition and subtraction of time intervals in minutes. Create a multi-step contextual problem involving addition and/or subtraction of time intervals in minutes, provide a solution, and explain using a visual model how to arrive at that solution.

Instructional Focus Statements

Level 3:

Students need to experience working with time in both analog and digital formats. It is important to note that students should be exposed to multiple manipulatives as they are learning time intervals including, but not limited to, a clock face and number lines. It is important to connect the similarities that exist in working with a clock face and in working with number lines. Students should be able to verbalize the similarities and differences of each as a manipulative for calculating elapsed time. While the length of time may be more than 60 minutes, unit conversion into hours is not an expectation at this grade level. This standard provides a great opportunity for students to make sense of problems (MP1) and represent them in multiple formats. Contextual problems should be written with the unknown in all positions and with a wide variety of problem types. See the Common Addition and Subtraction Situations Table embedded in the Tennessee Academic Standards.

Level 4:

The instructional focus should evolve so that students are experiencing contextual problems with increased rigor over time eventually encompassing situations with multiple time intervals with solutions that involve both addition and subtraction. Students should be able to supply valid, conceptually-based justifications for their thinking processes both verbally and in writing.

Standard 3.MD.A.2 (Major Work of the Grade)

Measure the mass of objects and liquid volume using standard units of grams (g), kilograms (kg), milliliters (ml), and liters (l). Estimate the mass of objects and liquid volume using benchmarks.

For example, a large paper clip is about one gram, so a box of about 100 large clips is about 100 grams.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Identify that liquid volume can be measured in milliliters or liters and vice versa. Identify that the mass of an object can be measured in grams or kilograms and vice versa.	Measure liquid volumes and masses of objects using standard units of grams, kilograms, and liters from a visual representation.	Measure liquid volumes using standard units of milliliters and liters. Measure the mass of objects using standard units of grams and kilograms. Estimate the mass of objects using benchmarks in a simple, real-world situation. Estimate liquid volume using benchmarks in a simple, real-world situation.	Estimate the mass of objects using benchmarks in a complex, real-world situation and provide a justification using appropriate mathematical vocabulary. Estimate liquid volume using benchmarks in a complex, real-world situation and provide a justification using appropriate mathematical vocabulary.

Instructional Focus Statements

Level 3:

The first focus of this standard revolves around a student's ability to read a scale in order to determine mass or liquid volume. For measuring mass, this will most often involve reading a digital scale. The challenge will be that as grams and kilograms are a part of the metric system, the scales will read mass typically to the nearest tenth of a unit. Students at this grade have not experienced decimal numbers. This is not the appropriate place to do so. Thus, the objects to be measured will have to be very carefully selected so that their mass equates to a whole number. If a different device is used that promotes fractional measures, the measurements need to be to the nearest half unit.   One important note, within the science standards, it is not necessary at this grade level for students to distinguish the difference between weight and mass. Thus, this distinction should not be drawn in mathematics either.

For liquid volume, students will be essentially reading a vertical number line in order to measure the liquid volume. The challenge here is that as liters and milliliters are also metric units, realistic measuring devices (e.g. graduated cylinders) will be marked off in tenths of a unit. We do not work with tenths in the grade 3 fraction standards and thus should not here as either. Thus, it is best if students are measuring to the nearest half unit.

The second focus of the standard revolves around estimating mass and liquid volume using benchmarks. It is imperative at this grade level that the benchmark be provided. Students have not developed a conceptual understanding of either mass or volume.   Thus, students have not interacted with either property enough to have developed the spatial sense to be able to estimate without a benchmark. The mathematical focus of this piece is using estimation to get to an application of multiplication.

<u>Level 4:</u>

Students should interact with estimations grounded with a benchmark measure with increasing rigor over time.

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Standard 3.MD.B.3 (Supporting Content)

Draw a scaled pictograph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled graphs.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Draw a pictograph with a scale of 1 to represent a data set with up to 4 categories. Draw a bar graph with a scale of 1 to represent a data set with up to 4 categories. Solve one-step "how many more" and "how many less" problems using information presented in a bar graph with a scale of 1.	Solve two-step "how many more" and "how many less" problems using information presented in bar graphs with a scale of 1. Complete a scaled picture graph to represent a data set with up to four categories. Complete a scaled bar graph to represent a data set with up to four categories.	Draw a scaled picture graph to represent a data set with several categories. Draw a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled pictographs with several categories.	Determine the scale of the graph, given a picture graph or bar graph. Collect data on four or more categories, choose an appropriate scale, justify the choice of scale, generate a pictograph or bar graph to represent the data, generate a one-step "how many more" question, a one step "how many less" question, and at least two 2-step questions that can be answered from the graph providing solutions to the created questions. Use data gathered from multiple graphs with multiple categories on each to answer two-step "how many more" and "how many less" problems. Additionally, the graphs need not be presented in the same way (i.e. one as a pictograph and 1 as a bar graph) or need not have the same scale.

Instructional Focus Statements

Level 3:

There are really three instructional priorities embedded within this standard for students: their ability to read data presented in a graph, their ability to create a graph, and their ability to answer questions from data presented in a graph.

As to reading data presented in a graph, there is a significant difference between grade 3 and what students have previously experienced. In grade 3, students draw picture graphs in which each picture represents more than one object. Likewise, students draw bar graphs in which the height of a given bar in tick marks must be multiplied by the scale factor in order to yield the number of objects in the given category. These developments connect with the emphasis on multiplication (standard 3.OA.3) in this grade.

Students need to generate picture graphs and bar graphs where each picture or tick mark represents more than one object. It is also important to expose students to gathering categorical data in authentic contexts, including contexts arising in their other classes (e.g., science and social studies). Another nice connection here is for students to tie in standard 3.G.A.1 where students categorize shapes and then they make a picture graph or bar graph to represent their data. That said, students do not have to generate the data every time they work on making bar graphs and picture graphs. After some experiences in generating the data, most work in producing bar graphs and picture graphs can be done by providing students with data sets.

Finally, students should be reading data from a graph in order to answer one- and two-step "how many more" and "how many less" problems. A good resource for guiding varying structure within these questions is the “Common Addition and Subtraction Situations” table located in the Tennessee Mathematics Standards. This work will support standard 3.OA.D.8.

Level 4:

Students should be exposed to data sets and graphs that increase in complexity over time. Additionally, students should progress to where they are able to assimilate data from multiple graphs in order to answer real-world and mathematical problems.

Standard 3.MD.B.4 (Supporting Content)

Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units: whole numbers, halves, or quarters.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Generate measurements of data by measuring lengths using rulers to the nearest whole unit. Represent the data by making a line plot, where the horizontal scale is marked off in whole number units.	Generate measurements of data by measuring lengths using rulers marked with whole numbers and halves of an inch. Represent the data by making a line plot, where the horizontal scale is marked off in whole number and half number units.	Generate measurements of data by measuring lengths using rulers marked with halves and fourths of an inch. Represent the data by making a line plot where the student must generate the appropriate horizontal markings for the scale of the line plot.	Generate measurements of data by measuring lengths using rulers marked with halves and fourths of an inch where there is a wide diversity of lengths being included in the set. Represent the data by making a line plot where the student must generate the appropriate horizontal markings for the scale of the line plot. Create several one and two-step “how many more” and “how many less” real-world problems that could be answered by looking at the data.

Instructional Focus Statements

Level 3:

Students should be using their developing knowledge of fractions and number lines to work with measurement data involving fractional measurement values in halves and fourths of an inch. This standard supports and gives an application for 3.NF.A.2. Students are being exposed to two key concepts within this standard: the ability to measure to the quarter inch and also to create a line plot to represent their data. To make a line plot from data which they have either generated or has been provided in a table, the student should be able to ascertain the greatest and least values in the data and self-create the line plot within those bounds. One important note, there is no need to sort the observations, or to count them, before producing the line plot.

Level 4:

In extending their understanding of this standard, students should be exposed to generating measurements from a wide diversity of lengths. Students can use data from a graph in order to create one- and two-step "how many more" and "how many less" problems. This work will support standard 3.OA.D.8.

Standard 3.MD.C.5 (Major Work of the Grade)
Recognize that plane figures have an area and understand concepts of area measurement.
3.MD.C.5a Understand that a square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area.
3.MD.C.5b Understand that a plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Cover a rectangular area with provided unit squares in a way such that they do not overlap and such that no gaps are created.	Identify a square with side length 1 unit as a “unit square”. Given a rectangular area pre-covered in unit squares, count how many squares were used but not necessarily make the connection that the number of squares represents the area of the rectangle.	Recognize a plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. Identify a square with side length 1 unit as a “unit square “and recognize that it can be used to measure area.	Explain why some unit squares are larger in size than others and explain why both are called “unit squares”. Explain why there can be no gaps or overlaps with unit squares when calculating area using appropriate mathematical vocabulary in either verbal or written form. Explain why squares are used to calculate area. Explain the differences between measuring area and measuring length.

Instructional Focus Statements

Level 3:

It is very important that students develop a conceptual understanding of area. Instruction on area should be integrated with the instruction of multiplication as students should be modeling multiplication using array models and tiling. This model is difficult for students to grasp if they have not been exposed to an introduction of the concept of area.

Students need the opportunity to grasp that they are measuring a two-dimensional bounded space by counting units of area represented by a square. They need to make the connection that just as length has a unit such as inches, area has a unit represented by a square such as square inches. Additionally, they should extend their understanding that when measuring length a ruler has no "gaps" and no "overlaps" to understand that when covering a two-dimensional space to measure area, there also should be no gaps or overlaps. Students should learn the terminology "unit square", understand what it means, and be able to properly use the terminology when describing area.

Level 4:

As students deepen their understanding of the concept of area, they should be able to provide explanations of the terminology used for area, why it is important that there are not gaps and overlaps when calculating area, and the similarities and differences between calculating area and calculating length. Students should do so in both written and verbal form using appropriate mathematical terminology.

Standard 3.MD.C.6 (Major Work of the Grade)
Measure areas by counting unit squares (square centimeters, square meters, square inches, square feet, and improvised units).

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
When provided a rectangular area completely covered by unit squares without gaps, accurately count the number of unit squares but not necessarily connect the number of squares to the concept of area.	When provided a rectangular area partially covered in unit squares without gaps, finish covering the rectangle with unit squares of the appropriate size and count the number of unit squares to determine area. When provided any shaped area completely covered by unit squares without gaps, accurately count the number of unit squares and connect the number of square to the concept of area.	Measure area of a regular plane figure by generating unit squares within the figure and then counting the unit squares. When given a visual representation of an irregular plane figure partially covered in unit squares, finish covering the shape with unit squares of the appropriate size and then determine area by counting the unit squares.	Measure areas of irregular plane figures by generating and counting unit squares.

Instructional Focus Statements

Level 3:

Students began developing this understanding in their work on standard 2.G.A.2. Instruction should build on this understanding and should center on continuing to develop a student's conceptual understanding of the concept of area and how it differs from length. The connection should be made that just as in measuring the length of an object where units (i.e. inches) can be counted, in area they can also count units which are represented by squares. Just as the unit of length remains constant, the size of the square must also remain constant. The focus needs to not only be on regular plane figures where all unit squares are provided, but also on students being able to generate the unit squares themselves. Additionally, students need to build an understanding that as the unit changes the size of the square also changes. Students should develop this understanding prior to learning that multiplying length by width will also produce the area of a rectangular region.

<u>**Level 4:**</u>

The focus of instruction should expand beyond working with regular plane figures to irregular plane figures where students are able to generate all of the unit squares, count them, and recognize that what they have found represents the area.

Standard 3.MD.C.7 (Major Work of the Grade)
Relate area of rectangles to the operations of multiplication and addition.
3.MD.C.7a Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths.
3.MD.C.7b Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning.
3.MD.C.7c Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a x b* and *a x c.* Use area models to represent the distributive property in mathematical reasoning. *For example, in a rectangle with dimensions 4 by 6, students can decompose the rectangle into 4 x 3 and 4 x 3 to find the total area of 4 x 6.* (See Table 3 - Properties of Operations)
3.MD.C.7d Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	**Students with a level 2 understanding of this standard will most likely be able to:**	**Students with a level 3 understanding of this standard will most likely be able to:**	**Students with a level 4 understanding of this standard will most likely be able to:**
Partition a rectangle into rows and columns of unit squares and find the total number of squares but not necessarily connect this activity to the concept of area. Split a rectangle into two smaller rectangles and calculate the area of each smaller rectangle either by multiplying or by tiling in isolation from one another. Decomposing rectilinear shapes into non-overlapping rectangles.	Find the area of a rectangle with whole number side lengths by tiling it. Multiply side lengths to find areas of rectangles with whole number side lengths in mathematical problems when a visual representation is provided. Divide a rectangle into two smaller rectangles in order to find the area and write this as (a x c) + (a x b) but not connect the decomposition to the distributive property.	Find the area of a rectangle with whole number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths. Multiply side lengths to find areas of rectangles with whole number side lengths in real-word and mathematical problems. Represent whole number products as rectangular areas. Use tiling to show a concrete case that the area of a rectangle with	Find the area of a rectangle with whole number side lengths by tiling it, show that the area is the same as would be found by multiplying the side lengths, and explain why the two are the same. Solve complex real-world problems involving whole number products and justify the mathematical reasoning for the solution using rectangular areas. Solve real world and mathematical problems involving the distributive property employing an area model

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Find areas of rectilinear figures when a pre-partitioned visual representation providing all measurements is provided.	whole number side lengths *a* and *b* + *c* is the sum of *a x b* and *a x c*. Use area models to represent the distributive property. Find areas of rectilinear figures when all measurements are provided by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts in mathematical and real world problems.	as the mathematical reasoning to justify the solution. Explain the connection between strategies for multiplication and using the distributive property as a strategy for decomposition of rectangular shapes to find area. Find areas of rectilinear figures with missing side lengths by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts in mathematical and real world problems.

Instructional Focus Statements

Level 3:

As students work with this set of standards, they should be progressing from the conceptual understanding built in standard 3.MD.C.6 around counting all unit squares, to connecting multiplying the number of rows by the number of unit squares in each row, to then a realization that area of a rectangular region can be found by multiplying length by width. If conceptual understanding is not built first and students are not given time to develop a true conceptual understanding of area, they will most likely struggle finding area of non-rectangular shapes.

Students must first learn to interpret measurement of rectangular regions as a multiplicative relationship between the number of square units in a row and the number of rows. This is built from the understanding developed using repeated addition in standard 2.OA.C.4. The ability to see and calculate area relies on the development of spatial structuring where students employee increasingly sophisticated strategies over time. The first of which tends to be skip-counting using the number of units in each row. This can lead to students eventually multiplying the number of units in each row by the number of rows. Simultaneously, students should be learning to partition a rectangle into identical squares forming the array by drawing line segments to form rows and columns. Finally, students should realize that the number of units can be found by multiplying the length by the width. Many activities that involve seeing and making arrays of squares to form a rectangle might be needed to build a robust conceptual understanding of area. Once students have a strong conceptual understanding of calculating area within rectangular regions, the areas presented should not all be rectangular but decomposable into rectangles. Once students make this transition, they may need to go back to tiling in order to calculate the area.

The final component of this standard is students using concrete objects or drawings along with their understanding of area to validate the distributive property. They understand and explain that the area of a rectangular region of, for example, 15 length-units by 5 length-units can be found either by multiplying 15 x 5, or by adding two products, e.g., 10 x 5 and 5 x 5, illustrating the distributive property. This should be closely tied to the instruction of standard 3.OA.B.5.

<u>Level 4:</u>

Students should be able to communicate the multiple connections that exist between the concept of area and multiplication in both verbal and written form. They should be exposed to real-world and mathematical problems that are increasingly rigorous over time. Additionally, Students should continue to develop their understanding of different strategies that can be used to find the area of complex shapes that can be decomposed into rectangles. Students should be able to see multiple ways to decompose the shapes leading to multiple ways to calculate area. One such strategy is for students to see these as rectangles that have been removed from a larger rectangle. In the example below, they see a larger rectangle with a smaller rectangle removed leading to a subtraction expression to calculate area as opposed to an addition one. Students should be able to make mathematical connections between the multiple expressions used to calculate area for a single figure which can be decomposed into rectangles.

Standard 3.MD.D.8 (Supporting Content)

Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Find the perimeter of a polygon when given a visual representation with all side lengths identified.	Find the perimeter of a triangle when given the lengths of all 3 sides. Find the perimeter of a square when given the length of 1 side.	Solve real-world and mathematical problems involving perimeters of polygons. Find an unknown side length for a polygon when provided the polygon's perimeter. Given a rectangle, choose a rectangle with the same area but a different perimeter. Given a rectangle, choose a rectangle with the same perimeter but a different area.	Given a rectangle, generate a rectangle with the same area but a different perimeter. Given a rectangle, generate a rectangle with the same perimeter but a different area. Explain the mathematical similarities and differences between area and perimeter.

Instructional Focus Statements

Level 3:

Perimeter is new terminology for grade 3 students. Conceptual understanding should be grounded in and built from the experiences students had solving contextual problems involving lengths in grade 2. Initially, it may be useful to pre-mark the sides of the polygons with unit length marks to mimic a ruler, allowing students to count the unit lengths much like they are encouraged to count unit squares when learning about area. It is important that students are counting the length-units and not the endpoints and that they understand why. Eventually, the lengths of the sides can be labeled with numerals and students can mark off unit lengths on each side of the polygon. The final step in conceptual understand is when a student no longer needs to mark off the unit lengths and can work with perimeter without them present. One of the most common instructional errors is to move too quickly away from the concrete markings to the abstract numeral before a student has really conceptually grasped what perimeter is. As students continue to work with rectangles, parallelograms, and regular polygons, they can discuss and justify faster ways to find the perimeter length than just adding all of the lengths. This should be presented as discovery learning as opposed to students being presented with a formula to memorize.

Once students have developed a strong conceptual understanding of both perimeter and area, students then begin to work on problems that differentiate their measure creating rectangles with the same area but different perimeters and rectangles with the same perimeter but different areas. This thought process is one of the earliest cornerstone for what will become the geometric modeling standards in high school courses.

Level 4:

Once students have developed a conceptual understanding of both area and perimeter and can verbalize the differences and similarities between them, instruction should shift to let students work with increasingly rigorous real-world and mathematical problems some of which illicit an understanding of perimeter, some and understanding of area, and some both area and perimeter. These real-world and mathematical problems should have hallmarks of mathematical modeling as students need opportunities to model with mathematics prior to being exposed to the modeling standards in high school.

Geometry (G)

Standard 3.G.A.1 (Supporting Content)

Understand that shapes in different categories may share attributes and that the shared attributes can define a larger category. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify examples of quadrilaterals and the subcategories of quadrilaterals. Identify examples and non-examples of a quadrilateral. Define the attribute that makes a polygon a quadrilateral.	Recognize examples of quadrilaterals that have shared attributes and that the shared attributes can define a larger category.	Recognize and sort examples of quadrilaterals that have shared attributes and show that the shared attributes can define a larger category. Draw examples of quadrilaterals that don't belong to the categories of rhombuses, rectangles, and squares. Draw examples of quadrilaterals with specific attributes.	Draw examples and non-examples of quadrilaterals and categorize them by specific attributes. Create a verbal and/or written description that represents a particular quadrilateral.

Instructional Focus Statements

Level 3:

In standard 2.G.A.1, students identified quadrilaterals. In grade 3, students should have a variety of experiences investigating quadrilaterals. Students should understand that all quadrilaterals have four sides (edges) and four vertices (corners) through investigation. Manipulatives such as toothpicks, straws, pipe cleaners and geoboards are tools that can allow students to explore the attributes of the varying quadrilaterals.

Students should have opportunities to draw and look at shapes (including using technology) to explore the attributes and defining characteristics of quadrilaterals. Based upon prior knowledge that a quadrilateral is a polygon with four sides and four vertices, students should be able to discuss and explain how quadrilateral shapes are alike and different. Students should be able to observe characteristics of the angles and the relationship between opposite sides to categorize quadrilaterals and non-quadrilaterals. Instruction should also focus on discovering the unique attributes that make a quadrilateral a rectangle, rhombus, and square. Additionally, students should be able to sort/classify shapes into more than one category of a quadrilateral. For example, a square is classified as both a rectangle and a rhombus.

Instructors should pose questions such as "Is a rhombus a quadrilateral?" or "Is a quadrilateral a rhombus?" Students should be able to explain their thinking and consider if it is always, sometimes or never true. Explanations should include precise mathematical vocabulary when describing the properties of quadrilaterals.

Level 4:

As students extend their understanding of quadrilaterals, they should be able to draw examples and non-examples and categorize them by specific attributes. By asking students to draw non-examples of quadrilaterals, they must reason about what defines a particular quadrilateral and explain what attributes belong to the specific category. As an extension, students should be able to create a verbal and/or written description that represents a particular quadrilateral.

Standard 3.G.A.2 (Supporting Content)
Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area and describe the area of each part as 1/4 of the area of the shape.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Partition circles and rectangles into two, three and four equal parts, describe the shares using the words halves, thirds, fourths, half of, a third of, and a fourth of. Recognize that equal parts of identical wholes do not have to be the same shape.	Partition shapes into two, three and four equal parts, describe the shares using the words halves, thirds, fourths, half of, a third of, and a fourth of.	Partition shapes into parts with equal areas and express the area as a unit fraction (with denominators of 2, 3, 4, 6, or 8) of the whole (Equal areas with the same shape). Partition shapes into parts with equal areas and express the area as a unit fraction (with denominators of 2, 3, 4, 6, or 8) of the whole (equal areas do not have to be the same shape).	Partition shapes in multiple ways into parts with equal areas and expresses the area as a unit fraction of the whole (Equal areas do not have to be the same shape). Determine the whole and identify the fraction, Given a partially partitioned shape with a shaded portion that represents a unit fraction. When given a fractional piece, construct the whole. For example, when given a piece that represents a third, create the whole from that third.

Instructional Focus Statements

Level 3:

In the grade 3, students will build on their understanding from standard 2.G.A.3 where they learned to partition a shape into 2, 3 or 4 equal parts and described the parts as halves, thirds and fourths. Standard 3.G.A.2 can be taught in conjunction with 3.NF.A.1 where students are learning to identify unit fractions and record fractional pieces in numerical form, which is new learning in third grade.

The focus of this standard is partitioning a shape into equal sized pieces and describing the equal parts as a unit fraction. In grade 3, students are limited to denominators of 2, 3, 4, 6, and 8. For example, students should be able to partition a shape into 6 equal sized pieces and describe each part as 1/6 of the whole.

As students look at the same shape partitioned in the same amount of equal sized pieces in different ways, they should understand that each equal sized piece is the same size piece of the whole. For example, when given 2 squares with one square vertically partitioned in 4 equal sized pieces and another same sized square horizontally portioned into 4 equal sized pieces, students should make the connection that each ¼ piece of the whole is the same size in both squares. Students will need many opportunities to explore how one of those pieces can be transformed into the differently shaped, same size piece. For example, this can be done by paper folding and partitioning of same-sized wholes to discover that, as long as the wholes are the same size, the shapes of the fraction piece can be different.

It is important that appropriate vocabulary is used and modeled throughout this exploration. For example, expressing each equal part as a unit fraction of the whole.

Level 4:

Allow students to explore shapes other than rectangles and circles to find multiple ways to represent the same unit fraction. Having students look critically at a shaped that has not been completely partitioned (ex. one side shows a half and the other side is partitioned into fourths) and asking them to represent a fractional part of an area involves determining the whole and then considering the size of the shaded part of the area in relation to the whole. This can be done by giving students a unit fraction piece or a non-unit fraction and create the whole from that piece. It also creates an opportunity for discussion about the importance of the size of the whole when discussing fractional pieces by giving them fourths from a variety of wholes and creating wholes using those fractional pieces.

Standard 3.G.A.3 (Supporting Content)
Determine if a figure is a polygon.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Distinguish between two-dimensional and three dimensional figures Distinguish between a line segment and a curve.	Sort figures into categories based on attributes that are the same and different. Determine if a figure is open or closed.	Determine if a figure is a polygon. Draw examples of polygons.	Explain why a figure is or is not a polygon using precise mathematical vocabulary. Draw and explain non-examples of polygons.

Instructional Focus Statements

Level 3:

In grades 1 and 2, students identified and illustrated two-dimensional shapes based on a shape's attributes such as determining the number of sides and the angles the polygon contains. This is the first time students will be introduced to the term polygon. Students should develop a conceptual understanding that a polygon it is a two-dimensional closed figure with three or more line segments.

Instruction should focus on creating generalizations about what students notice when analyzing polygons and non-polygons. For example, students should engage in sorting shapes based on what they noticed from an example the teacher may have posted of a polygon and not a polygon, rather than memorizing the formal definition. When students discover attributes of polygons it allows them to develop a deeper conceptual understanding of this geometric figure.

Level 4:

As students solidify why a shape is or is not a polygon, they should be able to explore and explain through reasoning of its attributes. A Frayer Model is a good tool for students to use to organize their thinking about the characteristics of examples and non-examples of polygons. Explanations of the student generated drawings should include precise mathematical vocabulary verbally and in writing.